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Effects of Additives in Finite Journal Bearing Using Finite Difference Method

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Abstract: In this paper general bearing considered and the effect of additives in lubrication of journal bearing. The generalized Reynolds equation for two layer fluid is derived in and is applied finite journal bearing. The finite modified Reynolds type equation is obtained for solved numerically by using FDM technique with a grid space of θ=9^{0}and $\Delta \bar{z}$ =0.05. The effect of two layer increases the pressure and increases load.

Index Terms: Peripheral layer thickness, Eccentricity ratio, Pre – load, Ratio of the viscosities

1 INTRODUCTION

In general, most of the lubricated systems can be considered to consist of moving (stationary) surfaces (plane/ curve or loaded/ unloaded) with a thin film of an external material (lubricant) between them. The presence of such a thin film between these surfaces not only helps to support considerable load but also minimizes friction. The characteristics such as pressure in the film, frictional force at the surface, flow rate of lubricant etc. of the system depend upon the nature of the surfaces, the nature of the lubricant film boundary conditions etc.

The equation governing the pressure generated in the lubricant film can be obtained by coupling the equation of motion with the equation of continuity and was first derived by Reynolds [15] and is known as "Reynolds Equation". In deriving this equation, the thermal effects, compressibility, viscosity variation, slip at the surface, inertia and surface roughness effects were ignored. Later this Reynolds equation is modified by including viscosity and density variation along the fluid film. Dowson [6] unified the various attempts in generalizing the Reynolds equation by considering the variation of fluid properties across as well as along the fluid film-thickness by neglecting slip effects at the bearing surfaces.

It may be noted that the effects of velocity-slip at the surface, is important on the flow behavior of gases and liquids particularly when the film-thickness is very small , the surface is very smooth [5].To study the effects of slip, Burgdorfer [3] modified the Reynolds equation for gas lubricated hydrodynamic bearings with "slip flow" under isothermal condition and pointed out that if $0<\frac{h}{\lambda}<1$ gas flow may be continuous and analysis can be carried out with modified slip boundary condition. Hsing and Malanoski [7] studied the effect of molecular mean free path in spiral-grooved thrust bearing and Tseng [18] used the Reynolds equation with slip to study the rarefaction effects of gas - lubricated bearings in the magnetic recording disk file.

MATHEMATICAL FORMULATION OF THE PROBLEM

The Physical configuration of the journal bearing is shown in fig (1.1). C be the clearance of the bearing, c = r-R and $\varepsilon=\frac{e}{c}$ be the eccentricity ratio as show in fig (1)

h is the total film thickness, is given by

$$h=c(1+\varepsilon\cos\theta)$$

$$\frac{\partial h}{\partial\theta}=-\varepsilon\sin\theta \tag{1}$$

The equation of governing to the fluid flow in the bearing is given by

$$\frac{\partial}{\partial x}\left[F\frac{h^3}{12\mu}\frac{\partial p}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{h^3}{12\mu}F\frac{\partial p}{\partial y}\right]=U\frac{\partial h}{\partial x} \tag{2}$$

$$F=\frac{(1-\frac{a}{h})^3(k-1)+1}{k} \tag{3}$$

The non-dimensional parameters are

x=Rθ, dx=Rdθ, $\bar{y}=\frac{y}{L} \Rightarrow y=\bar{y}L\ ,dy=Ld\bar{y}$

$$\bar{a}=\frac{a}{c}\ \Rightarrow\ \bar{h}=\frac{h}{c}\ ,\ \lambda=\frac{L}{2R} \tag{4}$$

$$\frac{\partial}{R\partial\theta}\left[F\frac{h^3}{12\mu}\frac{\partial p}{R\partial\theta}\right]+\frac{\partial}{L\partial\bar{y}}\left[\frac{h^3}{12\mu}F\frac{\partial p}{L\partial\bar{y}}\right]=\frac{U}{R}\frac{\partial h}{\partial\theta} \tag{5}$$

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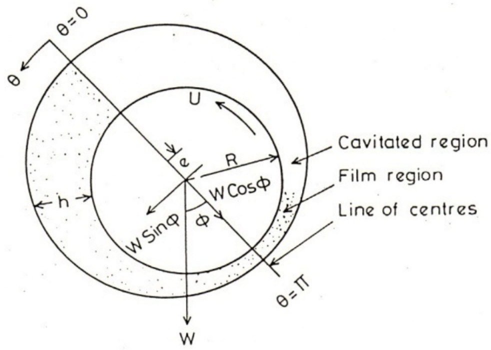


Fig 1: Journal bearing configuration

By substituting equation (4) in (5),then the modified Reynolds equation in a non dimensional form can be written as

$$\frac{\partial}{\partial\theta}\left[\bar{F}\frac{\bar{h}^3}{12\mu}\frac{\partial p}{\partial\theta}\right]+\frac{1}{4\lambda^2}\frac{\partial}{\partial\bar{y}}\left[\frac{h^3}{12\mu}\bar{F}\frac{\partial p}{\partial\bar{y}}\right]=RU\frac{\partial\bar{h}}{\partial\theta} \qquad (6)$$

where $\lambda^2=\dfrac{L^2}{4R^2}$

$$\bar{F}=\frac{(1-\frac{\bar{a}}{\bar{h}})^3(k-1)+1}{k} \qquad (7)$$

$$\bar{h}=c(1+\varepsilon\cos\theta)$$

By solving the above equation (6), we get the non-dimensional pressure as

$$\bar{p}=\frac{pc^2}{\mu uR} \qquad (8)$$

Now the equation (5) reduced to

$$\frac{\partial}{\partial\theta}\left[\bar{F}\bar{h}^3\frac{\partial\bar{p}}{\partial\theta}\right]+\frac{1}{4\lambda^2}\frac{\partial}{\partial\bar{y}}\left[\bar{F}\bar{h}^3\frac{\partial\bar{p}}{\partial\bar{y}}\right]=-12\varepsilon\sin\theta \qquad (9)$$

The boundary conditions for fluid film pressure are

$$\bar{p}=0 \ at \ \ \theta=0$$
$$\bar{p}=0 \ at \ \ \theta=\pi \qquad (10)$$

The modified Reynolds equation is solved numerically using Finite difference method .The film domain under consideration is divided by grid points as shown in fig (1.2.b). In finite increment format, the terms of equation (9) can be written as

$$\frac{1}{\Delta\theta}\left[\bar{F}_{i+\frac{1}{2},j}\left(\frac{\bar{p}_{i,j}-\bar{p}_{i-1,j}}{\Delta\theta}\right)\right]-\left[\bar{F}_{i-\frac{1}{2},j}\left(\frac{\bar{p}_{i+1,j}-\bar{p}_{i,j}}{\Delta\theta}\right)\right]+$$
$$\frac{1}{4\lambda^2}\frac{1}{\Delta\bar{y}}\left[\bar{F}_{i,j+\frac{1}{2}}\left(\frac{\bar{p}_{i,j}-\bar{p}_{i,j-1}}{\Delta\bar{y}}\right)-\left[\bar{F}_{i,j-\frac{1}{2}}\left(\frac{\bar{p}_{i,j+1}-\bar{p}_{i,j}}{\Delta\bar{y}}\right)\right]\right]=-12\varepsilon\sin\theta \qquad (11)$$

By solving we get as

		$\bar{f}_{i,j+\frac{1}{2}}$	
$\bar{p}_{i-1,j}$	$\bar{f}_{i-\frac{1}{2},j}$		$\bar{f}_{i-\frac{1}{2},j}$
		$\bar{p}_{i,j}$	
	$\bar{p}_{i,j-1}$	$\bar{f}_{i,j-\frac{1}{2}}$	

$\Delta\theta$

Fig 2: Grid point notation for film domain

$$\frac{1}{(\Delta\theta)^2}\left[\bar{F}_{i+1/2,j}\,\bar{p}_{i,j}-\bar{F}_{i+1/2,j}\,\bar{p}_{i+1,j}-\bar{F}_{i-1/2,j}\,\bar{p}_{i+1,j}+\bar{F}_{i-1/2,j}\,\bar{p}_{i,j}\right]+$$
$$\frac{1}{4\lambda^2}\frac{1}{\Delta\bar{y}^2}\left[\bar{F}_{i,j+1/2}\,\bar{p}_{i,j}-\bar{F}_{i,j+1/2}\,\bar{p}_{i,j-1}-\bar{F}_{i,j-1/2,}\,\bar{p}_{i+1,j}+\bar{F}_{i,j-1/2}\,\bar{p}_{i,j}\right]=-12\varepsilon\sin\theta$$

$$4\lambda^2\bar{r}^2\left[\bar{F}_{i+1/2,j}\,\bar{p}_{i,j}-\bar{F}_{i+1/2,j}\,\bar{p}_{i-1,j}-\bar{F}_{i-1/2,j}\,\bar{p}_{i+1,j}+\bar{F}_{i-1/2,j}\,\bar{p}_{i,j}\right]+$$
$$\left[\bar{F}_{i,j+1/2}\,\bar{p}_{i,j}-\bar{F}_{i,j+1/2}\,\bar{p}_{i,j-1}-\bar{F}_{i,j-1/2,}\,\bar{p}_{i,j+1}+\bar{F}_{i,j-1/2}\,\bar{p}_{i,j}\right]=-12\varepsilon4\lambda^2\sin\theta\,\Delta\bar{y}^2 \qquad (12)$$

By substituting this equation in (9), we get

$$\bar{p}_{i,j}\left[4\lambda^2\bar{r}^2\left(\bar{F}_{i+1/2,j}+\bar{F}_{i-1/2,j}\right)+\left(\bar{F}_{i,j+1/2}+\bar{F}_{i,j-1/2}\right)\right]=-12\varepsilon4\lambda^2\Delta\bar{y}^2\sin\theta+4\lambda^2\bar{r}^2\bar{F}_{i+1/2,j}\,\bar{p}_{i-1,j}$$
$$+4\lambda^2\bar{r}^2\bar{F}_{i-1/2,j}\,\bar{p}_{i+1,j}+\bar{F}_{i,j+1/2}\,\bar{p}_{i,j-1}+\bar{F}_{i,j-1/2}\,\bar{p}_{i,j+1}$$
$$c_0\bar{p}_{i,j}=4\lambda^2\bar{r}^2\bar{F}_{i+1/2,j}\,\bar{p}_{i-1,j}+4\lambda^2\bar{r}^2\bar{F}_{i-1/2,j}\,\bar{p}_{i+1,j}+\bar{F}_{i,j+1/2}\,\bar{p}_{i,j-1}+\bar{F}_{i,j-1/2}\,\bar{p}_{i,j+1}+-12\varepsilon4\lambda^2\Delta\bar{y}^2\sin\theta$$

$$\bar{p}_{i,j}=c_1\,\bar{p}_{i-1,j}+c_2\,\bar{p}_{i+1,j}+c_3\,\bar{p}_{i,j-1}+c_4\,c_{i,j+1}+c_5 \qquad (13)$$

The coefficient c_0, c_1, c_2, c_3, c_4, c_5, defined as

$$c_0=\left[4\lambda^2\bar{r}^2\left(\bar{F}_{i+1/2,j}+\bar{F}_{i-1/2,j}\right)+\left(\bar{F}_{i,j+1/2}+\bar{F}_{i,j-1/2}\right)\right] \qquad c_1=4\lambda^2\bar{r}^2\bar{F}_{i+1/2,j}/c_0$$

$$c_2=4\lambda^2\bar{r}^2\bar{F}_{i-1/2,j}/c_0 \qquad c_3=\bar{F}_{i,j+1/2}/c_0$$

$$\bar{r}=\frac{\Delta\bar{z}}{\Delta\bar{\theta}} \qquad c_4=\bar{F}_{i,j-1/2}/c_0$$

$$c_5=-48\lambda^2\Delta\bar{y}^2\varepsilon\sin\theta/c_0, \quad (14)$$

The pressure p is calculated numerically with grid spacing of

$$\Delta\bar{\theta}=0.05 \ and \ \ \Delta\bar{z}=9^0$$

The load carrying capacity of the bearing W, generated by the film pressure is obtained by

$$W=\int_{\theta=0}^{\theta=\pi}\int_{z=0}^{z=1/2}p\cos\theta\,d\theta.dz \qquad (15)$$

By using (9) in (14), we get non dimensional load as

$$\bar{W}=\frac{WC^2}{\mu UR}=\int_{\theta=0}^{\theta=\pi}\int_{z=0}^{z=1/2}p\cos\theta\,d\bar{\theta}.d\bar{z} \qquad (16)$$

$$\approx\bar{w}=\sum_{i=0}^{M}\sum_{j=0}^{N}\bar{p}_{ij}\,\Delta\bar{\theta}\ \Delta\bar{z} \qquad (17)$$

Where M+1 and N+1 are the grid point numbers in the x and z direction respectively.

RESULT AND DISCUSSION

The pressure in equation (13) the mesh of the film domain has 20 equal intervals along the bearing length and circumference. The coefficient matrix of the system of algebraic equation is of penta diagonal form. These equations have been solved by using sci-lab tools.

Pressure

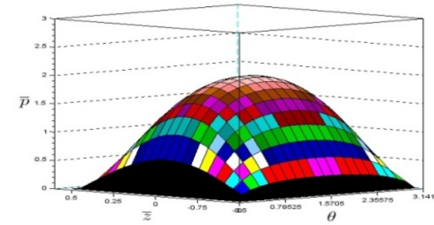
The variation of non-dimensional pressure $\overline{p}$ for different values of k with $\overline{a}$ =0.1and r=1.5 is shown in (1.3). It is observed that $\overline{p}$ increases for increasing value of k .Fig (4) shows The variation of film pressure $\overline{p}$ and different values of peripheral layer thickness $\overline{a}$ with k=0.5 and r=1.5.It is observed that $\overline{p}$ decreases for increasing values of $\overline{a}$

Load carrying capacity

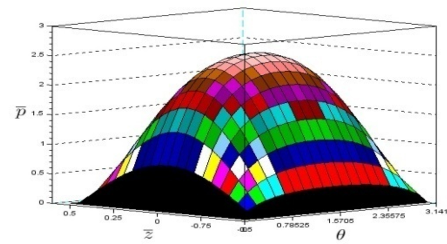
Fig (5) shows that the variation of non-dimensional load carrying capacity $\overline{W}$ with ε for different values of $\overline{a}$ at k=0.5. It is observed that the increasing values of $\overline{a}$ decrease the $\overline{W}$ and the corresponding values of load at different $\overline{a}$ are shown in table (1). Fig (6) shows that the variation of non-dimensional load carrying capacity $\overline{W}$ with ε for different values of $\overline{a}$ at k=1. It is observed that at k=1for any values of $\overline{a}$ there is no change in load carrying capacity and the corresponding values of load $\overline{W}$ at different $\overline{a}$ are shown in table (2).

Fig (7) shows that the variation of non-dimensional load carrying capacity $\overline{W}$ with ε for different values of $\overline{a}$ at k=1.5. It is observed that the increasing values of $\overline{a}$ decrease the $\overline{W}$ and the corresponding values of load $\overline{W}$ at different $\overline{a}$ are shown in table (3). Fig (8) shows that the variation of non-dimensional load carrying capacity $\overline{W}$ with ε for different values of k. It is observed that the increasing values of k increase the $\overline{W}$ and the corresponding values of load $\overline{W}$ are shown in table (4). Fig (9) shows that the variation of non-dimensional load carrying capacity $\overline{W}$ with $\bar{a}$ for different values of k. It is observed that the increasing values of k increase the $\overline{W}$ and the corresponding values of load $\overline{W}$ are shown in table (5).

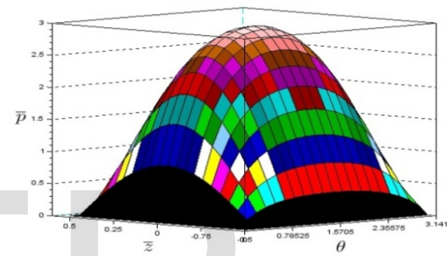
GRAPHS



(k=0.5)1.9849488

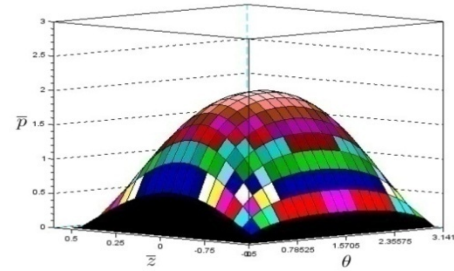


(k=1)2.5336184

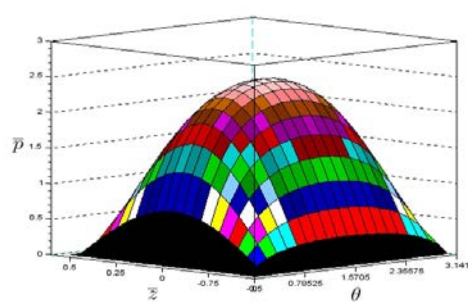


(k=2) 2.9452292

Fig 3: Non-Dimensionless pressure $\overline{p}$ for different values of k with a=0.1, r=1.5, epc=0.4, λ=0.75



($\overline{a}$=0.1) 1.9849488



($\overline{a}$=0.01)2.4582403

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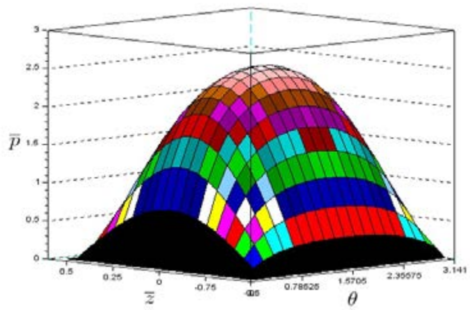

($\bar{a}$ =0.001)2.5257903

Fig 4: Non-Dimensionless pressure $\bar{p}$ for different values of $\bar{a}$ with k=2, r=1.5, epc=0.4, λ=0.75

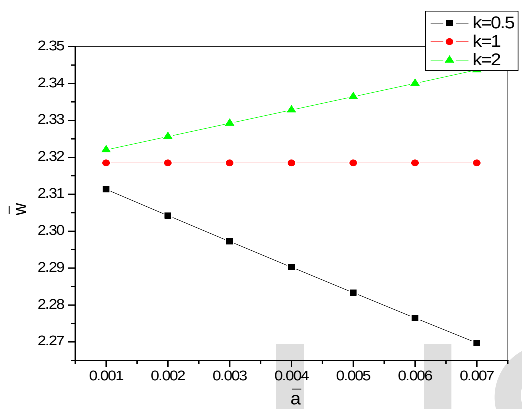

Fig 5 : Dimensionless load $\overline{W}$ Vs $\bar{a}$ for different k

Fig 7 : Dimensionless load $\overline{W}$ Vs $\bar{a}$ for different k

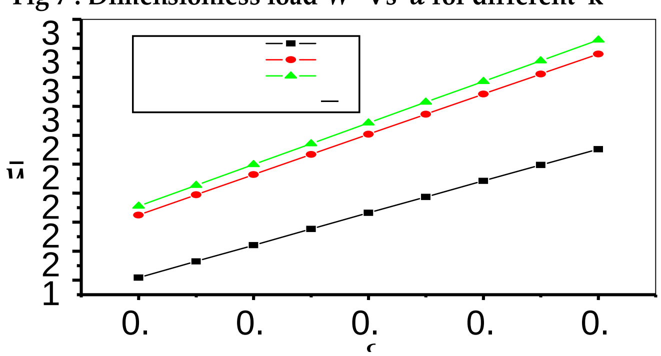

Fig 6: Dimensionless load $\overline{W}$ Vs ε for different $\bar{a}$ at k=0.5

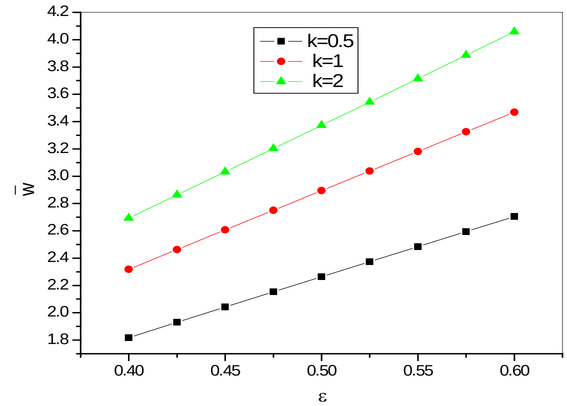

Fig 7: Dimensionless load $\overline{W}$ Vs ε for different values of k

Table (1): Dimensionless load $\overline{W}$ Vs Eccentricity Ratio ε for different $\bar{a}$

ε (k=0.5)	$\bar{a}$ =0.1	$\bar{a}$ =0.01	$\bar{a}$ =0.001
0.4	1.8179197	2.2496755	2.3113123
0.425	1.9299309	2.3893012	2.4550559
0.45	2.0416285	2.5287398	2.5986687
0.475	2.1529895	2.6679776	2.7421427
0.5	2.2639904	2.807	2.8854699
0.525	2.3746069	2.9457921	3.0286422
0.55	2.4848139	3.0843378	3.1716511
0.575	2.5945856	3.2226201	3.3144883
0.6	2.7038955	3.3606209	3.4571453

Table (2): Dimensionless load $\overline{W}$ Vs Eccentricity Ratio ε for different $\bar{a}$

ε(k=1)	$\bar{a}$ =0.1	$\bar{a}$ =0.01	$\bar{a}$ =0.001
0.4	2.3184555	2.3184555	2.3184555
0.425	2.4626796	2.4626796	2.4626796
0.45	2.6067802	2.6067802	2.6067802
0.475	2.7507501	2.7507501	2.7507501
0.5	2.894582	2.894582	2.894582
0.525	3.0382688	3.0382688	3.0382688
0.55	3.1818033	3.1818033	3.1818033
0.575	3.325178	3.325178	3.325178
0.6	3.468386	3.468386	3.468386

Table (3) : Dimensionless load $\overline{W}$ Vs Eccentricity Ratio ε for different $\bar{a}$

k=1.5	$\bar{a}$ =0.1	$\bar{a}$ =0.01	$\bar{a}$ =0.001
0.4	2.6934824	2.3545107	2.3220443

0.425	2.8629672	2.501163	2.46651
0.45	3.0327116	2.6477294	2.6108559
0.475	3.2027459	2.7942064	2.7550752
0.5	3.3731035	2.9405913	2.8991611
0.525	3.5438211	3.0868815	3.0431069
0.55	3.7149395	3.2330754	3.1869057
0.575	3.8865036	3.3791717	3.3305512
0.6	4.0585628	3.5251702	3.4740366

Table (4): Dimensionless load $\overline{W}$ Vs Eccentricity Ratio ε for different k

ε	K=0.5	K=1	K=2
0.4	1.8179197	2.3184555	2.6934824
0.425	1.9299309	2.4626796	2.8629672
0.45	2.0416285	2.6067802	3.0327116
0.475	2.1529895	2.7507501	3.2027459
0.5	2.2639904	2.894582	3.3731035
0.525	2.3746069	3.0382688	3.5438211
0.55	2.4848139	3.1818033	3.7149395
0.575	2.5945856	3.325178	3.8865036
0.6	2.7038955	3.468386	4.0585628

Table (5) : Dimensionless load $\overline{W}$ Vs peripheral layer thickness $\overline{a}$ for different k

ε	K=0.5	K=1	K=2
0.001	2.3113123	2.3184555	2.3220443
0.002	2.3042301	2.3184555	2.3256369
0.003	2.297208	2.3184555	2.3292332
0.004	2.2902452	2.3184555	2.3328332
0.005	2.2833412	2.3184555	2.336437
0.006	2.276495	2.3184555	2.3400444
0.007	2.2697061	2.3184555	2.3436555

SUMMARY

In this chapter general bearing considered and the effect of additives in lubrication of journal bearing. The generalized Reynolds equation for two layer fluid is derived and is applied finite journal bearing. The finite modified Reynolds type equation is obtained for solved numerically by using FDM technique with a grid space of θ=9⁰and $\Delta\overline{z}$=0.05. The effect of two layer increases the pressure and increases load

NOMENCLATURE

a	peripheral layer thickness
p	pressures
h	Film thickness
μ	Viscosity of the fluid
x,y,z	Cartesian coordinates
θ	Circumference angle
c	Clearance
e	Eccentricity
R	Radius of the shaft
K	Ratio of viscosity near the surface to the purely hydrodynamic
u, v,w	Velocity component of the film in x,y,z direction
U	Velocity
ε	Eccentricity ratio

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